P.O.C.A. WONG SIU CHING SECONDARY SCHOOL
PURE MATHEMATICS
ALGEBRA : MATHEMATICAL INDUCTION
ASSIGNMENT 3

Date	Name	Grade / Score
		/15

1. Let us consider the following problem about the population of rabbits. Let a_n be the number of pairs of rabbits in the *n*th month. We also assume that $a_1 = 1$, that is there is a pair of young rabbits in the first month. We are known that the initial pair of rabbits gave birth to a new pair of rabbits per month, and that after two months each new pair behaved similarly.

   (a) Write down the number of pairs of rabbits in the first ten months. (2 marks)

   (b) Find a relation between a_n, a_{n-1} and a_{n-2} for $n \geq 3$. (2 marks)

   (c) Show by induction that $a_n = \dfrac{\alpha^n - \beta^n}{\alpha - \beta}$, where $\alpha > \beta$ are the roots $x^2 - x - 1 = 0$. (5 marks)

   This sequence was discovered by ***Fibonacci (1175-1250)*** in connection with a problem about rabbits. This sequence was so-called the ***Fibonacci sequence***.

2. Let $P(n)$ and $Q(n)$ be two propositions involving positive integers n satisfying :

(1) $P(1)$ is true,
(2) if $P(n)$ is true for some positive integer n then $Q(n)$ is also true, and
(3) if $Q(n)$ is true for some positive integer n then $Q(n+1)$ is also true.

Find the set of values of n so that $P(n)$ is true and the set of values of n so that $Q(n)$ is true.
Explain briefly. (3 marks)

3. Let $P(m, n)$ be proposition involving positive integers m and n satisfying the following properties :

(1) $P(1, n)$ is true for all positive integer n,
(2) $P(m, 1)$ is true for all positive integer m, and
(3) if $P(i, j+1)$ and $P(i+1, j)$ are true for some positive integer i and j then $P(i+1, j+1)$ is also true.

Find the set of values of (m, n) so that $P(m, n)$ is true.
Explain briefly. (3 marks)