P.O.C.A. WONG SIU CHING SECONDARY SCHOOL
PURE MATHEMATICS
ALGEBRA I : BINOMIAL THEOREM
ASSIGNMENT 5

Date	Name	Grade / Score
		/15

1. Let $(1+x+x^2)^n = \sum_{k=0}^{2n} a_k x^k$.

(a) Evaluate (5 marks)

(i) $\sum_{k=0}^{2n} a_k$.

(ii) $\sum_{k=0}^{2n} (-1)^k a_k$.

(iii) $\sum_{k=0}^{n} a_{2k}$ and $\sum_{k=1}^{n} a_{2k-1}$.

(iv) $\sum_{k=1}^{2n} k a_k$.

(b) Replacing x by $\frac{1}{x}$, show that $a_{2n-k} = a_k$. (2 marks)

(c) Using the identity $(1+x+x^2)^n(1-x+x^2)^n = (1+x^2+x^4)^n$ and (b), show that $\sum_{k=0}^{2n} (-1)^k {a_k}^2 = a_n$.

(4 marks)

(d) By considering the expansion of $(1-x^3)^n$, find the value of $\sum_{k=0}^{r}(-1)^k\binom{n}{k}a_{r-k}$, for the case r is not a multiple of 3 and the case if $r=3m$ for some positive integer m. (4 marks)