| Date | Name | Grade / Score |
|---|---|---|
| | | **/15** |

1.    Let $P_n(x) = \cos(n\cos^{-1}x)$, where $-1 \le x \le 1$ and $n$ is a non-negative integer.            (8 marks)

(a)    Prove that $P_{n+1}(x) = 2xP_n(x) - P_{n-1}(x)$.

(b)    Using induction to show that $P_n(x)$ is polynomial in $x$ of degree $n$ with leading coefficient $2^{n-1}$, for $n \ge 1$.

2. Given that $f(x) = 4x^4 - 2x^3 - 16x^2 + 5x + 9$ and $g(x) = 2x^3 - x^2 - 5x + 4$. Find the G.C.D. $d(x)$ of $f(x)$ and $g(x)$ and find polynomials $u(x)$ and $v(x)$ such that $d(x) = u(x)f(x) + v(x)g(x)$. (8 marks)