

**P.O.C.A. WONG SIU CHING SECONDARY SCHOOL**  
**PURE MATHEMATICS**  
**ALGEBRA : ALGEBRAIC INEQUALITIES**  
**ASSIGNMENT 9B**

| Date | Name | Grade / Score |
|------|------|---------------|
|      |      | <b>/15</b>    |

1. (a) Let  $x > 0$  and  $k$  be a positive integer. Prove that

$$(k+1)(x-1) \leq x^{k+1} - 1 \leq (k+1)x^k(x-1),$$

and that the equality holds if and only if  $x = 1$ .

- (b) Hence, prove that, if  $y > 1$  then  $y^{k+1} - (y-1)^{k+1} < (k+1)y^k < (y+1)^{k+1} - y^{k+1}$ .  
(Hint : Put  $x = \frac{y}{y-1}$  and then  $x = \frac{y+1}{y}$ ).

(c) Deduce that  $\frac{n^{k+1}}{k+1} < 1^k + 2^k + \cdots + n^k < \frac{(n+1)^{k+1} - 1}{k+1}$ .

2. Let  $x, y$  be two positive real numbers and  $m, n$  two positive integers. Prove that

$$x^m y^n + x^n y^m \leq x^{m+n} + y^{m+n},$$

and that the equality holds if and only if  $x = y$ .