

P.O.C.A. WONG SIU CHING SECONDARY SCHOOL
PURE MATHEMATICS
CALCULUS : INFINITE SEQUENCES
ASSIGNMENT 11B

Date	Name	Grade / Score
		/15

1. Let a_1, b_1, c_1 be positive numbers whose sum is 1 and for $n \geq 1$, define

$$\begin{cases} a_{n+1} = a_n^2 + 2b_n c_n \\ b_{n+1} = b_n^2 + 2c_n a_n, \\ c_{n+1} = c_n^2 + 2a_n b_n \end{cases}$$

where $a_n \geq b_n \geq c_n$.

(a) Show that $a_n + b_n + c_n = 1$ for all $n \geq 1$. (2 marks)

(b) Show that $\{a_n\}$ is decreasing and $\{c_n\}$ is increasing. (4 marks)

(c) Show that $a_{n+1} - c_{n+1} \leq (a_n - c_n)^2$, for $n \geq 1$.

(4 marks)

Hence deduce that $a_{n+1} - c_{n+1} \leq (a_1 - c_1)^{2^n}$.

(d) Show that $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ all converge to a same limit.

(5 marks)

Find this limit.