

P.O.C.A. WONG SIU CHING SECONDARY SCHOOL
PURE MATHEMATICS
CALCULUS : INFINITE SERIES
ASSIGNMENT 12

Date	Name	Grade / Score
		/15

1. **Comparison Test.** Let $0 \leq a_k \leq b_k$ for all k . Show that if $\sum_{k=1}^{\infty} b_k$ converges then so is $\sum_{k=1}^{\infty} a_k$.

Using the comparison test to show that $\sum_{k=0}^{\infty} \frac{1}{k!}$ converges.

2. Let $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ for $n \geq 1$.

(a) Show that if k is a positive integer then $\frac{1}{2^{k-1} + 1} + \frac{1}{2^{k-1} + 2} + \cdots + \frac{1}{2^k} \geq \frac{1}{2}$.

(b) Let m be the largest positive integer such that $2^m \leq n$. Show that $a_n \geq 1 + \frac{m}{2}$.

(c) Deduce that $\lim_{n \rightarrow \infty} a_n = \infty$.