| Date | Name | Grade / Score |
|------|------|---------------|
|      |      | /15           |

1.  Discuss the continuity of $f(x) = \begin{cases} \frac{1}{2}x + 5 & \text{if } x \le 0 \\ x + 85 & \text{if } x > 0 \end{cases}$ at 0.  (3 marks)

2.  Let $f(x) = \begin{cases} 0, & x \le 0 \\ a + b\cos px, & x \in (0,1) \\ 1, & x \ge 1 \end{cases}$. Find $a$ and $b$ in terms of $p$ such that $f$ is continuous everywhere.  (5 marks)

3. Suppose $f : [0,1] \to [0,1]$ is continuous. Prove that there is some $\alpha \in [0,1]$ such that $f(\alpha) = \alpha$. (4 marks)

4. Let $f$ be a function satisfying $|f(x) - f(y)| \le |x - y|$ for all $x, y \in \mathbf{R}$. Show that $f$ is continuous on $\mathbf{R}$. (3 marks)