2020-
POST MOCK 2
MATH CP

PAPER 1

P.O.C.A. WONG SIU CHING SECONDARY SCHOOL

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2020

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8:30 am – 10:45 am (2¼ hours)

This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Please stick the barcode label here.

Student Number							

Question No.	Marks
1 – 2	
3 – 4	
5 – 6	
7	
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Section A(1) (35 marks)

1. Simplify $\frac{x^{-4}y^5}{(x^2y)^3}$ and express your answer with positive indices. (3 marks)

2. (a) Round off 135.79 to 1 decimal place.
   (b) Round up 135.79 to 1 significant figure.
   (c) Round down 135.79 to the nearest integer.

(3 marks)

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3. Factorize

(a) $9x^2 - 4y^2$,

(b) $9x^2 - 4y^2 - 4y - 6x$.

(3 marks)

4. There are some balls in a bag, including 6 red balls, some blue balls and some white balls. It is given that the numbers of blue balls and white balls are in the ratio $2:3$ and the probability of drawing a blue ball from the bag is $\frac{2}{7}$. Find the total number of balls in the bag. (4 marks)

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5. Consider the formula $x + y = \frac{4x - y + 1}{3}$.

(a) Make x the subject of the above formula.

(b) If the value of y is increased by 1, write down the change in the value of x.

(4 marks)

6. (a) Find the range of values of x which satisfy both $\frac{2x-6}{3} \leq 4(x+2)$ and $6 - 3x > 0$.

(b) How many integers satisfy both the inequalities in (a).

(4 marks)

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7. Stanley invested \$100 000 in Bank A at a simple interest rate $r\%$ per annum for 3 years. He gets a simple interest of \$30 000 afterwards.

(a) Find r.

(b) The amount received from Bank A is then invested in Bank B at an interest rate $r\%$ per annum, compounded quarterly, for 1 year. Find the interest received from Bank B, correct to the nearest dollar.

(4 marks)

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8. Figure 1 shows the distribution of the districts in which students of a school live. It is known that the number of students living in New Territories East is more than that living in Kowloon by 42.

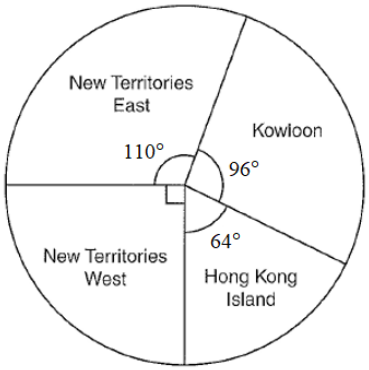

Figure 1

(a) Find the number of students in the school.

(b) After the first semester, some students living in Kowloon leave the school. Is it possible that the angle of the sector representing the students living in Kowloon be halved? Explain your answer.

(5 marks)

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9. It is given that $f(x)$ partly varies directly as x and partly varies directly as x^2. Suppose that $f(4) = 24$ and $f(-3) = 45$.

(a) Find $f(x)$.

(b) Let $A(a, 9)$ and $B(b, 9)$ be two distinct points lying on the graph $y = f(x)$. Find the distance between A and B.

(5 marks)

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Section A(2) (35 marks)

10. Let $f(x) = x^3 + g(x)$, where $g(x)$ is a quadratic polynomial. When $f(x)$ is divided by $(x+2)(x-3)$ and when $f(x)$ is divided by $(x-3)^2$, the remainders are $x-k$ and $kx-21$ respectively.

(a) Find the value of k. (3 marks)

(b) Find $g(x)$. (4 marks)

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11. The following shows the number of feedbacks given to the passages posted on a bulletin board.

13, 15, 22, 35, 35, 35, 41, 44, 44, 54, 56, 60, 65, 78, 93

(a) Find the mean, median and the inter-quartile range of the number of feedbacks. (3 marks)

(b) The 2 passages with the fewest feedbacks are deleted from the bulletin board. The administrator then posts 2 new passages on the board. If the mean number of feedbacks is increased by 5 and the median and the inter-quartile range remain unchanged, find the number of feedbacks given to the 2 new passages. (4 marks)

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12. Figure 2 shows the travel graph of Paul and Henry from town A to town C (via town B) on a day. Paul drives at a constant speed throughout the journey. Henry takes a rest after reaching town B. Then, Henry's driving speed is twice that of Paul when he drives from town B to town C.

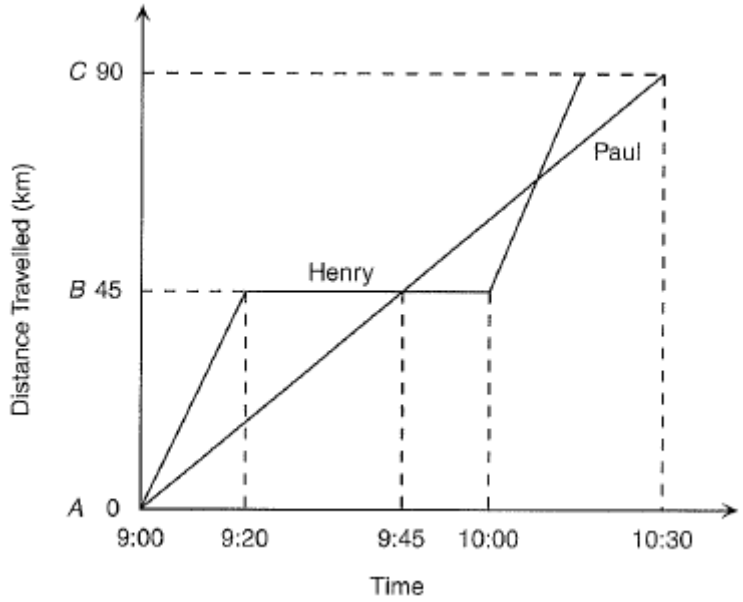

Figure 2

(a) Find the distance travelled by Paul during the period which Henry is taking rest. (2 marks)

(b) After Paul and Henry meet at town B, at what time do they meet again? (3 marks)

(c) Henry claims that if he takes rest for 10 more minutes, he will reach town C later than Paul. Is his claim correct? Explain your answer. (2 marks)

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13. Figure 3(a) shows a sheet of paper $ABCD$, which is formed by cutting the sector OBC away from the sector OAD. It is given that $AB = BO = 8\text{ cm}$ and $\angle AOD = 135°$. By joining AB and DC together, $ABCD$ is folded to form a mould. The mould is used to make cupcakes. After baking, the cupcake is in the shape formed by a frustum of a right circular cone and a hemisphere as shown in Figure 3(b).

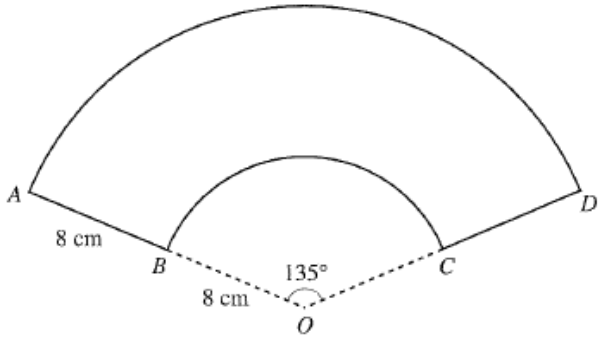

Figure 3(a)

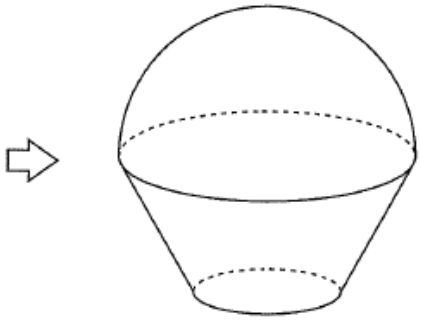

Figure 3(b)

(a) Find the radius of the hemisphere. (2 marks)

(b) Find the volume of the cupcake in terms of π. (5 marks)

(Leave your answer in the simplest surd form if necessary.)

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14. In Figure 4, $ABCDE$ is a circle. BD and CE intersect at M. $\angle CAM = \angle DAE$.

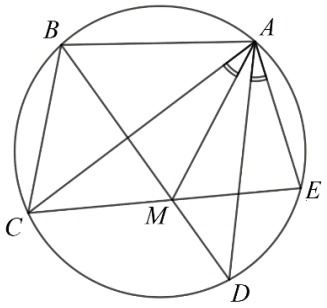

Figure 4

(a) Prove that $\angle CBM = \angle MAE$. (2 marks)

(b) Suppose ΔMAB is an isosceles triangle with $MA = MB$. Prove that

(i) $BA // CE$,

(ii) $\Delta BCM \cong \Delta AEM$.

(5 marks)

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Section B (35 marks)

15. The graph in Figure 5 shows the linear relation between $\log_9 x$ and $\log_{27} y$. The slope and the intercept on the horizontal axis of the graph are $\frac{1}{4}$ and -4 respectively. Express the relation between x and y in the form $y = Ax^k$, where A and k are constants. (3 marks)

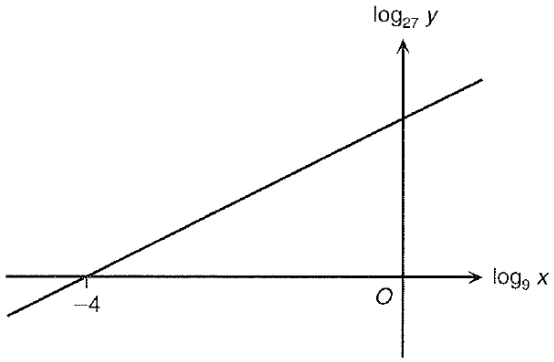

Figure 5

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16. There are 3 classical songs, 5 popular songs and 4 rock songs in a playlist. 4 songs are selected from the list to play. Find the number of possible play sequences if

(a) no classical songs are selected; (1 mark)

(b) at least one song from each genre is selected. (2 marks)

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17. The circles $C_1 : x^2 + y^2 - 18x - 14y + 105 = 0$ and $C_2 : x^2 + y^2 - 6x - 8y + 21 = 0$ intersect at two distinct points A and B. C_3 is another circle passing through A and B and the point $(5, 6)$.

(a) (i) Show that $x^2 + y^2 - 18x - 14y + 105 + k(x^2 + y^2 - 6x - 8y + 21) = 0$, where $k \neq -1$, is also a circle passing through A and B.

(ii) Hence, or otherwise, find the equation of C_3.

(5 marks)

(b) Let O be the origin and P be a moving point on C_3. M is a point on OP such that $OM : MP = 2 : 3$. Find the equation of the locus of M when P moves. (3 marks)

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18. Figure 6 shows a building with a horizontal base ABC such that $AB = BC = CA = 10\text{ m}$. P, Q and R are points vertically above A, B and C respectively such that $PA = 30\text{ m}$, $QB = 20\text{ m}$ and $RC = 10\text{ m}$.

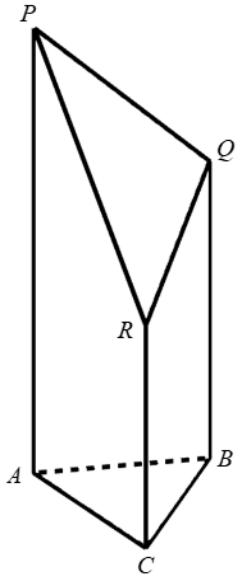

Figure 6

(a) Show that PQR is an isosceles triangle. (2 marks)

(b) Find $\tan\angle APR$ and $\cos\angle APQ$. (2 marks)

(c) Find the angle between the planes PQR and $APRC$. (6 marks)

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19. In Figure 7, $OABC$ is a rectangle lying on the rectangular coordinate plane with $OA = 8\sqrt{2}$ units and $OC = 8$ units. P and Q start moving at the same time. P is moving from O to A with a constant speed of $\sqrt{2}$ unit/second while Q is moving from C to O with a constant speed of 1 unit/second. Let t (in second) be the time elapsed since P and Q started moving, where $0 < t < 8$.

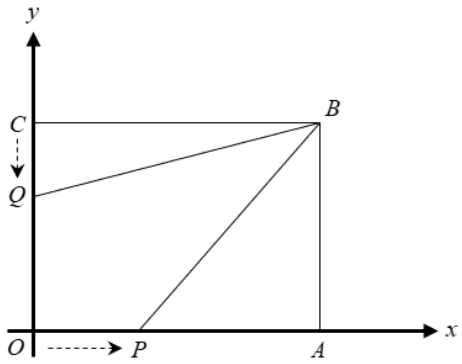

Figure 7

(a) Express the length of PA and CQ in terms of t. Hence show that the area of quadrilateral $OPBQ$ is a constant. (3 marks)

(b) Suppose P has moved to a position such that $OP = PA$. A parabola Γ passes through the points P, B and C. A vertical line $L : x = h$ cuts QB, PB and Γ at H, M and N respectively.

(i) Find the equation of Γ.

(ii) Show that $MN = -\frac{1}{4}h^2 + 3\sqrt{2}h - 16$.

(iii) Using the method of completing square, find the maximum length of MN.

(iv) When MN attains its maximum, find area of pentagon $OPMHQ$: area of ΔMBH.

(8 marks)

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