Secondary 6
Post Mock 2
2019-20
Marking Scheme

MARKING SCHEME

Solution	Marks	Remarks
1. $\dfrac{x^{-4}y^5}{(x^2y)^3}$		
$=\dfrac{x^{-4}y^5}{x^6y^3}$	1M	for $(ab)^m = a^m b^m$ or $(a^m)^n = a^{mn}$
$=\dfrac{y^{5-3}}{x^{6+4}}$	1M	for $\dfrac{c^p}{c^q} = c^{p-q}$ or $c^{-p} = \dfrac{1}{c^p}$
$=\dfrac{y^2}{x^{10}}$	1A	
	----------(3)	
2. (a) 135.8	1A	
(b) 200	1A	
(c) 135	1A	
	----------(3)	
3. (a) $9x^2 - 4y^2$		
$= (3x+2y)(3x-2y)$	1A	
(b) $9x^2 - 4y^2 - 4y - 6x$		
$= (3x+2y)(3x-2y) - 2(2y+3x)$	1M	for using the result of (a)
$= (3x+2y)(3x-2y-2)$	1A	
	----------(3)	
4. Let $2k$ and $3k$ be the number of blue balls and white balls respectively.	1M	
$\dfrac{2k}{6+2k+3k} = \dfrac{2}{7}$	1M+1A	
$14k = 12 + 10k$		
$4k = 12$		
$k = 3$		
Thus, there are $6 + 2(3) + 3(3) = 21$ balls in the bag.	1A	
	----------(4)	

MARKING SCHEME

Solution	Marks	Remarks
5. (a) $x+y=\frac{4x-y+1}{3}$		
$3x+3y=4x-y+1$	1M	
$-x=-4y+1$	1M	for putting x on one side
$x=4y-1$	1A	
(b) Increase by 4	1A	
	----------(4)	
6. (a) $\frac{2x-6}{3}\le 4(x+2)$		
$2x-6\le 12x+24$		
$-10x\le 30$	1M	for putting x on one side
$x\ge -3$	1A	
$6-3x>0$		
$-3x>-6$		
$x<2$		
Thus, $-3\le x<2$.	1A	
(b) 5	1A	
	----------(4)	
7. (a) $100\ 000\cdot r\%\cdot 3=30\ 000$	1M	
$r=10$	1A	
(b) The interest received		
$=130\ 000\left(1+\frac{10\%}{4}\right)^{1\times 4}-130\ 000$	1M	
$=\$13\ 496$	1A	
	----------(4)	

MARKING SCHEME

Solution	Marks	Remarks
8. (a) The total number of students in the school		
$=\dfrac{42}{110-96}\times 360$	1M	
$=1080$	1A	
(b) Let n be the number of students leave the school.		
$\dfrac{1080\cdot\frac{96}{360}-n}{1080-n}=\dfrac{48}{360}$	1M+1A	
$\dfrac{288-n}{1080-n}=\dfrac{48}{360}$		
$103\ 680-360n=51\ 840-48n$		
$-312n=-51\ 840$		
$n=166.1538462$		
Since 166.1538362 is not an integer.		
Thus, it is impossible.	1f.t.	
	----------(5)	
9. (a) Let $f(x)=k_1x+k_2x^2$, $k_1,k_2\neq 0$	1M	
$\begin{cases}k_1(4)+k_2(4)^2=24\\k_1(-3)+k_2(-3)^2=45\end{cases}$	1M	for either one
Solving, we have $k_1=-6$ and $k_2=3$.		
Thus, $f(x)=-6x+3x^2$.	1A	
(b) Note that a and b are the roots of $-6x+3x^2=9$.	1M	
$3x^2-6x-9=0$		
$x^2-2x-3=0$		
$(x+1)(x-3)=0$		
$x=-1$ or 3		
Thus, the distance between A and B is $3-(-1)=4$.	1A	
	----------(5)	

MARKING SCHEME

	Solution	Marks	Remarks
10. (a)	Note that $f(x)=(x+2)(x-3)r(x)+x-k=(x-3)^2 s(x)+kx-21$ for some polynomials $r(x)$ and $s(x)$.	1M	for either one
	Therefore, $f(3)=3-k=3k-21$	1M	
	Thus, $k=6$.	1A	
		----------(3)	
(b)	Since the coefficient of x^3 in $f(x)$ is 1.		
	Let $f(x)=(x-3)^2(x-a)+6x-21$	1M	
	Therefore, $f(2)=2-6=(2-3)^2(2-a)+6(2)-21$	1M	
	Solving, we have $a=-3$.	1A	
	Hence, $f(x)=(x-3)^2(x+3)+6x-21$ $=x^3-3x^2-3x+6$		
	Thus, $g(x)=-3x^2-3x+6$.	1A	
		----------(4)	
11. (a)	$\text{Mean}=46$	1A	
	$\text{Median}=44$	1A	
	$\text{Inter-quartile range}=60-35=25$	1A	
		----------(3)	
(b)	Let a and b be the number of feedbacks given to the 2 new passages, where $a\le b$. $\dfrac{690-13-15+a+b}{15}=46+5$		
	$a+b=103$	1M	
	Since the median remains unchanged, $a\le 44$.	1M	
	Since the inter-quartile range remains unchanged and the lower quartile must be 35, $b\le 60$.	1M	
	Thus, $\begin{cases}a=44\\b=59\end{cases}$ or $\begin{cases}a=43\\b=60\end{cases}$.	1A	
		----------(4)	

MARKING SCHEME

Solution	Marks	Remarks
12. (a) The distance travelled by Paul		
$=\frac{90}{90}\times 40$	1M	
$=40$ km	1A	
	----------(2)	
(b) Let x minute be the time passed till they meet again after 9:00.		
y km be the distance travelled.		
$\begin{cases} \frac{y-0}{x-0}=\frac{90-0}{90-0} \\ \frac{y-45}{x-60}=\frac{90-0}{90-0}\times 2 \end{cases}$	1A+1A	
Solving, we have $x=75$ and $y=75$.		
Thus, they meet at 10:15.	1A	
	----------(3)	
(c) Let a minute be the time passed till Henry reach town C after 9:00.		
$\frac{90-45}{a-60}=\frac{90-0}{90-0}\times 2$		
$a=82.5$	1A	
Since Henry only arrives earlier than Paul by 7.5 minutes.		
Thus, his claim is correct.	1f.t.	
	----------(2)	

MARKING SCHEME

Solution	Marks	Remarks
13. (a) Let r cm be the radius of the hemisphere.		
$2\pi(8+8)\cdot\frac{135}{360}=2\pi r$	1M	
$r=6$		
Thus, the radius is 6 cm.	1A	
	----------(2)	
(b) Let h cm be the height of the circular cone.		
$h^2+6^2=(8+8)^2$	1M	
$h=2\sqrt{55}$		
Volume of the frustum $=\frac{1}{3}\pi(6)^2\left(2\sqrt{55}\right)\left[1-\left(\frac{8}{8+8}\right)^3\right]$	1M+1M	1M for volume of cone 1M for ratio of similar
$=21\sqrt{55}\pi$		
Volume of the hemisphere $=\frac{4}{3}\pi(6)^3\times\frac{1}{2}$	1M	for volume of hemisphere
$=144\pi$		
Thus, the volume of the cupcake is $\left(144+21\sqrt{55}\right)\pi$ cm^3 .	1A	
	----------(5)	

MARKING SCHEME

Solution	Marks	Remarks
14. (a) $\angle CBM = \angle CAD$ ($\angle$s in the same segment)		
$= \angle CAM + \angle MAD$		
$= \angle DAE + \angle MAD$ (given)		
$= \angle MAE$		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	----------(2)	
(b) (i) $\angle ABC + \angle BCE$		
$= (\angle ABM + \angle CBM) + \angle BCE$		
$= (\angle BAM + \angle CBM) + \angle BCE$ (base $\angle$s, isos. Δ)		
$= (\angle BAM + \angle MAE) + \angle BCE$ (proved in (a))		
$= \angle BAE + \angle BCE$		
$= 180°$ (opp. $\angle$s, cyclic quad.)		
Thus, $BA // CE$. (int. $\angle$s supp.)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
(ii) $\angle CMB = \angle MBA$ (alt. $\angle$s, $BA // CE$)		
$= \angle MAB$ (base $\angle$s, isos. Δ)		
$= \angle AME$ (alt. $\angle$s, $BA // CE$)		
$MB = MA$ (given)		
$\angle CBM = \angle MAE$ (proved in (a))		
Thus, $\Delta BCM \cong \Delta AEM$. (ASA)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	----------(5)	

MARKING SCHEME

Solution	Marks	Remarks
15. $\dfrac{\log_{27} y - 0}{\log_9 x + 4} = \dfrac{1}{4}$	1M	
$\log_{27} y = \dfrac{1}{4}\log_9 x + 1$		
$\dfrac{\log y}{\log 27} = \dfrac{1}{4}\dfrac{\log x}{\log 9} + 1$		
$\dfrac{\log y}{3\log 3} = \dfrac{1}{4}\dfrac{\log x}{2\log 3} + 1$	1M	
$\log y = \dfrac{3}{8}\log x + 3\log 3$		
$\log y = \log x^{\frac{3}{8}} + \log 27$		
$\log y = \log 27x^{\frac{3}{8}}$		
Thus, $y = 27x^{\frac{3}{8}}$.	1A	
	----------(3)	
16. (a) The number of possible ways		
$= P_4^{5+4}$		
$= 3024$	1A	
	----------(1)	
(b) The number of possible ways		
$= \left(C_2^3 C_1^5 C_1^4 + C_1^3 C_2^5 C_1^4 + C_1^3 C_1^5 C_2^4\right) \times 4!$	1M	for either two cases correct
$= 6480$	1A	
	----------(2)	

MARKING SCHEME

Solution	Marks	Remarks
17. (a) (i) Let $A(x_1, y_1)$ and $B(x_2, y_2)$.		
Since both C_1 and C_2 pass through A and B.		
We have $\begin{cases} x_1^2 + y_1^2 - 18x_1 - 14y_1 + 105 = 0 \\ x_1^2 + y_1^2 - 6x_1 - 8y_1 + 21 = 0 \\ x_2^2 + y_2^2 - 18x_2 - 14y_2 + 105 = 0 \\ x_2^2 + y_2^2 - 6x_2 - 8y_2 + 21 = 0 \end{cases}$.		
Then we have		
$\begin{cases} x_1^2 + y_1^2 - 18x_1 - 14y_1 + 105 + k(x_1^2 + y_1^2 - 6x_1 - 8y_1 + 21) = 0 \\ x_2^2 + y_2^2 - 18x_2 - 14y_2 + 105 + k(x_2^2 + y_2^2 - 6x_2 - 8y_2 + 21) = 0 \end{cases}$.		
Hence, the equation passes through A and B.	1M	for showing the equation passes through A and B
The equation can be rewrite into the form		
$(1+k)x^2 + (1+k)y^2 - (18+6k)x - (14+8k)y + 105 + 21k = 0$,	1M	for showing the equation is a circle
which is an equation of circle since $k \neq -1$.		
Thus, the equation is a circle passing through A and B.	1f.t.	
(ii) Put $(5, 6)$ into the equation in (a)		
$5^2 + 6^2 - 18(5) - 14(6) + 105 + k[5^2 + 6^2 - 6(5) - 8(6) + 21] = 0$	1M	
$-8 + 4k = 0$		
$k = 2$		
Thus, the equation of C_3 is		
$x^2 + y^2 - 18x - 14y + 105 + 2(x^2 + y^2 - 6x - 8y + 21) = 0$		
$3x^2 + 3y^2 - 30x - 30y + 147 = 0$		
$x^2 + y^2 - 10x - 10y + 49 = 0$	1A	
	----------(5)	
(b) Let $P(a, b)$ and $M(x, y)$		
$x = \dfrac{a \cdot 2 + 0 \cdot 3}{2+3} = \dfrac{2}{5}a$	1M	for either one
$y = \dfrac{b \cdot 2 + 0 \cdot 3}{2+3} = \dfrac{2}{5}b$		
Hence, $P\left(\dfrac{5}{2}x, \dfrac{5}{2}y\right)$.		
Since P is a moving point on C_3.		
$\left(\dfrac{5}{2}x\right)^2 + \left(\dfrac{5}{2}y\right)^2 - 10\left(\dfrac{5}{2}x\right) - 10\left(\dfrac{5}{2}y\right) + 49 = 0$	1M	
$\dfrac{25}{4}x^2 + \dfrac{25}{4}y^2 - 25x - 25y + 49 = 0$		
$25x^2 + 25y^2 - 100x - 100y + 196 = 0$		
Thus, the required equation is $25x^2 + 25y^2 - 100x - 100y + 196 = 0$.	1A	
	----------(3)	

MARKING SCHEME

Solution	Marks	Remarks
18. (a) $PQ=\sqrt{10^2+(30-20)^2}=10\sqrt{2}$	1M	for either one
$QR=\sqrt{10^2+(20-10)^2}=10\sqrt{2}$		
Since $PQ=QR$.		
Thus, PQR is an isos. Δ.	1f.t.	
	----------(2)	
(b) $\tan\angle APR=\dfrac{10}{30-10}=\dfrac{1}{2}$	1A	
$\cos\angle APQ=\dfrac{30-20}{10\sqrt{2}}=\dfrac{\sqrt{2}}{2}$	1A	
	----------(2)	
(c) Let M be a point on PR such that $QM\perp PR$.		
N be a point on PA such that $NM\perp PR$		
Then, the required angle is $\angle QMN$.	1M	
$PM=\dfrac{1}{2}PR=\dfrac{1}{2}\sqrt{10^2+(30-10)^2}=5\sqrt{5}$		
$QM=\sqrt{PQ^2-PM^2}=\sqrt{\left(10\sqrt{2}\right)^2-\left(5\sqrt{5}\right)^2}=5\sqrt{3}$	1A	
$MN=PM\tan\angle NPM=\left(5\sqrt{5}\right)\left(\dfrac{1}{2}\right)=\dfrac{5\sqrt{5}}{2}$	1M	
$PN=\sqrt{PM^2+MN^2}=\sqrt{\left(5\sqrt{5}\right)^2+\left(\dfrac{5\sqrt{5}}{2}\right)^2}=\dfrac{25}{2}$		
$QN=\sqrt{PQ^2+PN^2-2(PQ)(PN)\cos\angle QPN}$		
$=\sqrt{\left(10\sqrt{2}\right)^2+\left(\dfrac{25}{2}\right)^2-2\left(10\sqrt{2}\right)\left(\dfrac{25}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right)}=\dfrac{5\sqrt{17}}{2}$	1M	
Note that $QM^2+MN^2=\left(5\sqrt{3}\right)^2+\left(\dfrac{5\sqrt{5}}{2}\right)^2$		
$=\dfrac{425}{4}$	1M	
$=\left(\dfrac{5\sqrt{17}}{2}\right)^2$		
$=QN^2$		
Thus, the required angle is $90°$.	1A	
	----------(6)	

MARKING SCHEME

Solution	Marks	Remarks
19. (a) $PA = 8\sqrt{2} - \sqrt{2}t$ $CQ = t$	1A	
Area of quadrilateral $OPBQ = 8\sqrt{2} \cdot 8 - \frac{1}{2} \cdot 8\sqrt{2} \cdot t - \frac{1}{2} \cdot 8 \cdot \left(8\sqrt{2} - \sqrt{2}t\right)$	1M	
$= 32\sqrt{2}$		
Thus, the area of quadrilateral $OPBQ$ is always $32\sqrt{2}$.	1f.t.	
	----------(3)	
(b) (i) Note that P must be the vertex of Γ .		
Let $\Gamma : y = \left(x - 4\sqrt{2}\right)^2$	1M	
Put $C(0,8)$ into Γ . $8 = a\left(0 - 4\sqrt{2}\right)^2$ $a = \frac{1}{4}$		
Thus, $\Gamma : y = \frac{1}{4}\left(x - 4\sqrt{2}\right)^2$	1A	
(ii) Equation of PB is $\frac{y-0}{x-4\sqrt{2}} = \frac{8-0}{8\sqrt{2}-4\sqrt{2}}$ $y = \sqrt{2}x - 8$		
Hence, we have $M\left(h, \sqrt{2}h - 8\right)$ and $N\left(h, \frac{1}{4}\left(h - 4\sqrt{2}\right)^2\right)$.	1A	for either one
Thus, $MN = \left(\sqrt{2}h - 8\right) - \frac{1}{4}\left(h - 4\sqrt{2}\right)^2$ $= \sqrt{2}h - 8 - \frac{1}{4}\left(h^2 - 8\sqrt{2}h + 32\right)$ $= \sqrt{2}h - 8 - \frac{1}{4}h^2 + 2\sqrt{2}h - 8$		
$= -\frac{1}{4}h^2 + 3\sqrt{2}h - 16$	1f.t.	
(iii) $MN = -\frac{1}{4}h^2 + 3\sqrt{2}h - 16$ $= -\frac{1}{4}\left(h^2 - 12\sqrt{2}h\right) - 16$		
$= -\frac{1}{4}\left[h^2 - 12\sqrt{2}h + \left(6\sqrt{2}\right)^2 - \left(6\sqrt{2}\right)^2\right] - 16$	1M	
$= -\frac{1}{4}\left[\left(h - 6\sqrt{2}\right)^2 - 72\right] - 16$ $= -\frac{1}{4}\left(h - 6\sqrt{2}\right)^2 + 2$		
Thus, the maximum length of MN is 2.	1A	

MARKING SCHEME

Solution	Marks	Remarks
(iv) When MN attains its maximum, $h = 6\sqrt{2}$.		
Therefore, $M\left(6\sqrt{2}, 4\right)$		
Equation of QB is $\dfrac{y-4}{x-0} = \dfrac{8-4}{8\sqrt{2}-0}$		
$y = \dfrac{\sqrt{2}}{4}x + 4$		
Therefore, $H\left(6\sqrt{2}, 7\right)$.		
Area of $\Delta MBH = \dfrac{1}{2}\cdot(7-4)\cdot\left(8\sqrt{2}-6\sqrt{2}\right)$	1M	
$= 3\sqrt{2}$		
Thus, the required ratio is $\left(32\sqrt{2}-3\sqrt{2}\right):3\sqrt{2} = 29:3$.	1A	
	----------(8)	